HornStr's following work

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Abstract

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Introduction

Definition 1 (Unification Unary Predicates) *Consider a set of uninterpreted unary predicates:*

$$V_1(x), V_2(x), V_3(x), \dots \quad (1)$$

Instead of treating them separately, we introduce a single predicate $V(x)$ and encode the original predicate indices using an additional prefix:

$$V_1(x) \to V(1,x), V_2(x) \to V(2,x), \dots V_i(x) \to V(i,x)$$

where each predicate $V_i(x)$ is mapped to $V(p,x)$, where p is a unique identifier such as a character (e.g., $1, 2, \cdots, i$). This transformation ensures that all unary predicates are represented by a single function while preserving their original distinctions.

Definition 2 (Encoding of k-ary Uninterpreted Predicates (1)) *For predicates of higher arity, such as a binary predicate:*

$$V(x,y), \quad (2)$$

we transform it into a unary predicate by concatenating its arguments:

$$W(x\#y), \quad (3)$$

where '$\#$' represents a separator that distinguishes different arguments in the transformed representation. This encoding reduces the complexity of CHC solving by converting multi-argument predicates into a single-argument form.

Definition 3 (Encoding of k-ary Uninterpreted Predicates (2)) *An alternative approach to argument concatenation is encoding predicates using a synchronous product. Instead of:*

$$W(x\#y), \quad (4)$$

we may define W based on the synchronous product of x and y, preserving their structure while enabling parallel interpretation. However, this transformation requires rewriting all CHC clauses accordingly, which may not always be feasible.

A possible example of this encoding is the product of automata. The synchronous product of two sequences:

$$a_1.a_2.a_3 \dots \quad // \quad (b_1.b_2 \dots) \quad (5)$$

can be rewritten as:

$$(a_1, b_1).(a_2, b_2).(a_3, b_3) \quad (6)$$

where the new alphabet is the product of the initial alphabets. Some remapping might be required so that classical word representations can still be used effectively.